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ON THE STABILITY OF STEADY TEMPERATURE DISTRIBUTION IN A TRANSPARENT SOLID WITH INTERNAL ENERGY SOURCE

S. SELAMOGLU

Middle East Technical University, Ankara, Turkey

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Abstract—The stability of the steady solution of general heat conduction equation with temperature-dependent as well as position-dependent internal energy source, radiation, and physical properties is studied. It is shown by way of a simple example that a temperature-dependent energy source may cause the temperature to increase without limit. In case of locally one-dimensional steady-state temperature and property distributions, the condition for the temperature distribution in an infinite solid to be stable is derived. The result is discussed as to its validity in different cases.

NOMENCLATURE

C_k, constant coefficient, equation (9a);
c, specific heat;
$\mathbf{F}$, total radiative flux vector, equation (28a);
f, function of $\mathbf{r}$;
k, separation constant;
l, separation constant; distance;
l_c, characteristic length;
m, constant;
p, function of z;
Q, rate of internal energy generation per unit volume;
Q_0, constant;
q, function of z;
R, net rate of energy loss per unit volume (radiative);
$\mathbf{r}$, position vector;
r, function of z; cylindrical or spherical coordinate;
$\mathbf{S}$, energy source flux vector, equation (28b);
T, temperature;
T^*, perturbation temperature;
T_0, constant;
$\bar{T}$, dimensionless temperature, equation (2c);
t, time;
X, function of x;
x, rectangular coordinate;
Y, function of y;
y, rectangular coordinate; function of z;
Z, function of z;
z, rectangular coordinate; variable;
α, coefficient of diffusion, equation (23);
$\boldsymbol{\beta}$, vector function, equation (24);
$\boldsymbol{\beta}_1$, vector function, equation (26);
β, magnitude of $\boldsymbol{\beta}$;
γ, function, equation (25);
γ_1, function, equation (27);
θ, function of t;
κ, coefficient of thermal conductivity;
Λ, parameter;
λ, constant;
μ, separation parameter;
ξ, dimensionless coordinate, equation (2a);
ρ, density;
τ, dimensionless time, equation (2b);
χ, constant, equation (9b).

Subscripts

k, running index;
min, minimum value;
n, running index;

0, initial value; particular value;
s, steady value;
w, value at the boundaries.

1. INTRODUCTION

IN PROBLEMS of heat conduction through solids with or without internal energy generation or radiation the interest is frequently in finding the steady temperature distribution. When the energy source and the physical properties of the solid are independent of temperature the differential equation to be solved is usually manageable. It possesses, under certain conditions, a unique solution which is also the limit as time tends to infinity of the solution of the corresponding unsteady problem.

The heat conduction equation has been treated in varying degree of generality. In the relatively recent works [1–3], the energy source term was assumed, most generally, to be position- and time-dependent. The thermal properties of the medium were taken either constant [1–3] or position-dependent [4]. In some cases [5, 6], the heat conduction problem is considered with temperature-dependent properties but without energy source and radiation.

When the energy source and/or physical properties are temperature-dependent it cannot be claimed that the steady solution is always the physical limit of the corresponding unsteady solution. In those cases, an effort toward obtaining the steady solution from the steady equation alone may be successful but worthless since this solution may not represent a physically possible temperature distribution unless it is stable under an arbitrary perturbation. The situation will later be made more clear by way of a simple example.

It may formally be possible to judge the stability of the steady solution after solving the unsteady problem. This, however, is not always practical; one should be able to determine the stability from the very equation of heat transfer. Physically, the unsteady equation of transfer governs the readjustment of the local temperature at any instant in order to allow for the net gain in local internal energy due to various mechanisms, namely thermal conduction, energy generation, and radiation. When the latter are independent of temperature the attainment of a final steady-state temperature distribution is possible, so that the net energy flux across any closed control surface within the solid vanishes. On the other hand, when the mechanisms of energy input are strongly temperature-dependent, while the temperature tries to adjust itself to the local momentary state, it may be so that the rate of energy input increases more rapidly than the conduction and radiation losses could possibly follow. As a consequence, there may be no asymptotic limit for the temperature distribution. The temperature increases until the initial assumptions on the physics of the problem become invalid and new phenomena come into effect to alter them.

In the following, a simple demonstrative example is given firstly. Later, the heat conduction equation is considered in a form as general as practicable and its steady solution is analyzed from the stability point of view. The stability criterion in a locally one-dimensional problem, which is the outcome of the analysis, is discussed last.

2. AN EXAMPLE

Consider the one-dimensional, unsteady problem with constant material properties and with energy generation which is linear in temperature. The differential equation for the temperature is then

$$\frac{1}{\alpha}\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2} + \frac{1}{\kappa}(Q_0 + \lambda T), \tag{1}$$

where α is thermal diffusivity, κ thermal conductivity, Q_0 and λ are constants. The boundary conditions will be assumed, for simplicity, as constant temperature T_w at the two boundaries $x = 0$ and $x = l$. The initial condition is an arbitrary temperature distribution $T(x, 0) = T_0(x)$ provided that it is integrable within the interval $(0, l)$, and that it equals T_w at the boundaries.

The formulation may be made dimensionless by the following transforms:

$$x = \frac{l}{\pi}\xi, \quad t = \frac{l^2}{\pi^2 \alpha}\tau, \quad T = T_w + \frac{Q_0}{\lambda}\bar{T}. \quad \text{(2a, b, c)}$$

The resulting equation is

$$\frac{\partial \bar{T}}{\partial \tau} = \frac{\partial^2 \bar{T}}{\partial \xi^2} + \frac{\lambda}{\kappa}\frac{l^2}{\pi^2}\bar{T} + \frac{\lambda}{\kappa}\frac{l^2}{\pi^2}\left(1 + \frac{\lambda}{Q_0}T_w\right). \quad (3)$$

If, further, the last term is assumed to vanish,

$$1 + \frac{\lambda}{Q_0}T_w = 0, \quad (4)$$

the equation becomes a simple one,

$$\frac{\partial \bar{T}}{\partial \tau} = \frac{\partial^2 \bar{T}}{\partial \xi^2} + \frac{\lambda}{\kappa}\frac{l^2}{\pi^2}\bar{T} \quad (5)$$

with the conditions imposed on $\bar{T}(\xi, \tau)$

$$\bar{T}(0, \tau) = \bar{T}(\pi, \tau) = 0, \quad (6)$$

$$\bar{T}(\xi, 0) = \bar{T}_0(\xi), \quad \bar{T}_0(0) = \bar{T}_0(\pi) = 0. \quad (7)$$

The solution to equation (5) may be given as a trigonometric series [7]

$$\bar{T}(\xi, \tau) = \sum_{k=1}^{\infty} C_k \, e^{(\chi - k^2)\tau} \sin k\xi \quad (8)$$

where

$$C_k = \frac{2}{\pi}\int_0^\pi \bar{T}_0(\xi) \sin k\xi \, d\xi, \quad \chi = \frac{\lambda}{\kappa}\frac{l^2}{\pi^2}. \quad \text{(9a, b)}$$

The coefficient C_k exists, in general, for all values of parameter k. Since the least value of k is unity the temperature distribution is stable (i.e. it does not grow in time) if $\chi < 1$. In other words, $\bar{T}$ tends to the limit zero everywhere as time increases if

$$\lambda < \frac{\kappa \pi^2}{l^2}. \quad (10)$$

If, on the other hand, λ does not satisfy the above condition the temperature grows without limit. In the particular case where

$$\lambda = \frac{\kappa \pi^2}{l^2} \quad (11)$$

all components of $\bar{T}$ excepting the first will eventually die out leaving the steady distribution

$$\bar{T}_s = C_1 \sin \xi. \quad (12)$$

If, by chance, the initial distribution were to coincide exactly with one of the sine functions, say $\bar{T}_0(\xi) = T_0 \sin m\xi$, where T_0 is a constant and m is an integer, then one would have

$$\bar{T}(\xi, \tau) = T_0 e^{(\chi - m^2)\tau} \sin m\xi. \quad (13)$$

The stability of (13) cannot, however, be judged through a comparison between χ and m^2. Although it would appear that when $\chi < m^2$, even if $\chi > 1$, the solution tends to zero everywhere, this is not always the case. After a slight perturbation of $\bar{T}$ from the pure sinusoidal form instability sets in if $\chi > 1$.

Consider now the steady equation

$$\frac{d^2 \bar{T}_s}{d\xi^2} + \chi \bar{T}_s = 0, \quad (14)$$

subject ot the same homogeneous boundary conditions as (6). The problem always possesses a trivial solution

$$\bar{T}_s(\xi) = 0, \quad (15)$$

and a nontrivial solution

$$\bar{T}_s(\xi) = T_0 \sin m\xi \quad (16)$$

only when $\chi = m^2$ where m is an integer. From the previous discussion it is seen that both the trivial and nontrivial solutions are unstable if $\chi > 1$, and stable if $\chi = 1$. When χ is not an integer there is only the trivial solution which is unstable if $\chi > 1$ and stable otherwise.

Summarizing, the stability of the steady solution depends only on the value of χ. The existence of a solution of the steady equation does not guarantee its stability. In simple cases where the unsteady solution can be found without difficulty the steady solution can always be obtained from the unsteady solution provided that the latter assumes a limit as time tends to infinity. When this is difficult or not possible the stability of the steady solution should first be investigated before attempting to find it.

In the foregoing example it is interesting to note that, when the interval $(0, l)$ extends to infinity, the stability condition (10) reduces to

$$\lambda \leqslant 0, \qquad (10a)$$

which is more stringent than (10). One can introduce a characteristic length

$$l_c = \sqrt{(\kappa/\lambda)} \qquad (17)$$

so that when $l \gg l_c$ the medium may be considered as being of infinite extent, and when l is comparable to l_c the finite dimension of the medium must be taken into account. A reduction in l improves stability. However, this effect should depend upon the type of boundary conditions imposed, in general.

3. PERTURBATION EQUATION

In order to obtain a sufficiently general formulation of the problem the following assumptions are in order.

(a) The material is a homogeneous and isotropic solid. The physical properties may depend upon temperature as well as on coordinates in a differentiable manner.

(b) Energy is generated at a rate which may vary with temperature and coordinates in a differentiable manner.

(c) The material is transparent to thermal radiation so that volume emission and absorption take place. Emission may be interpreted as internal energy loss and absorption as energy source so far as their roles in the equation are concerned. They should both be obtained, in general, through integral manipulations extended over space, frequency, and temperature at any instant. It will suffice, however, in the present analysis to introduce a term which implicitly represents a temperature- and position-dependent energy loss whether this be due to radiative transfer or due to a different mechanism. The finding of the explicit temperature dependence of this loss term should be formidable in case of radiation.

Under the preceding assumptions the unsteady heat transfer equation may be written as

$$\rho c \frac{\partial T}{\partial t} = \nabla . (\kappa \nabla T) + Q - R \qquad (18)$$

where ρ, c, κ, are the density, specific heat, and the coefficient of thermal conductivity of the solid obeying Fourier law, Q is the rate of internal energy generation per unit volume, and R is the rate of net energy loss from unit volume, all being functions of position, $\boldsymbol{r}$, and temperature, T. The temperature is a function of position and time, t.

Let a steady temperature distribution $T_s(\boldsymbol{r})$ exist. The various quantities entering equation (18) attain steady values which will be indicated by subscript s [e.g. $\kappa_s = \kappa(\boldsymbol{r}, T_s)$, etc.]. They must satisfy the steady heat transfer equation

$$\nabla . (\kappa_s \nabla T_s) + Q_s - R_s = 0. \qquad (19)$$

The difference between (18) and (19) is simply the perturbation equation for the temperature distribution which is perturbed from its steady state;

$$\rho c \frac{\partial}{\partial t}(T - T_s) = \nabla . (\kappa \nabla T - \kappa_s \nabla T_s) + (Q - Q_s) - (R - R_s). \qquad (20)$$

Introducing $T^* = T - T_s$, the perturbation temperature, and linearizing the equation for small perturbations one can obtain

$$\rho_s c_s \frac{\partial T^*}{\partial t} = \kappa_s \nabla^2 T^* + \left[\left(\frac{\partial \kappa}{\partial T}\right)_{T=T_s} \nabla T_s + \nabla \kappa_s\right] \times \nabla T^* + \left\{\nabla . \left[\left(\frac{\partial \kappa}{\partial T}\right)_{T=T_s} \nabla T_s\right] + \left(\frac{\partial Q}{\partial T}\right)_{T=T_s} - \left(\frac{\partial R}{\partial T}\right)_{T=T_s}\right\} T^*. \qquad (21)$$

The coefficients of $T^*, \nabla T^*$ and $\nabla^2 T^*$ are known functions of position since they are all evaluated at the steady temperature T_s which is supposed to be known.

One can write equation (21) also in the form

$$\frac{1}{\alpha}\frac{\partial T^*}{\partial t} = \nabla^2 T^* + \boldsymbol{\beta} . \nabla T^* + \gamma T^* \qquad (22)$$

where $\alpha, \boldsymbol{\beta}, \gamma$ are functions of position only, given by

$$\alpha = \frac{\kappa_s}{\rho_s c_s} \text{(coefficient of thermal diffusivity)}, (23)$$

$$\boldsymbol{\beta} = \frac{1}{\kappa_s}(\kappa_s' \nabla T_s + \nabla \kappa_s) = (\nabla \ln \kappa)_s, \qquad (24)$$

$$\gamma = \frac{1}{\kappa_s}[\nabla . (\kappa_s' \nabla T_s) + Q_s' - R_s']. \qquad (25)$$

In the last three expressions prime denotes differentiation with respect to temperature [e.g. $\kappa_s' = (\partial\kappa/\partial T)_s$]. By the use of (19) in (25), γ may be expressed in a different form for later use;

$$\gamma = \nabla(\ln \kappa)_s' . \nabla T_s + \left(\frac{Q - R}{\kappa}\right)_s' \qquad (25a)$$

There is an alternative way in obtaining of the perturbation equation. If use is made of the flux vectors for radiation loss and energy source terms the perturbation equation becomes

$$\frac{1}{\alpha}\frac{\partial T^*}{\partial t} = \nabla^2 T^* + \boldsymbol{\beta}_1 . \nabla T^* + \gamma_1 T^* \quad (22a)$$

with

$$\boldsymbol{\beta}_1 = \frac{1}{\kappa_s}[(\nabla\kappa)_s + S_s' - F_s'], \qquad (26)$$

$$\gamma_1 = \frac{1}{\kappa_s}[\nabla . (\kappa_s' \nabla T_s) + \nabla . S_s' - \nabla . F_s'], \qquad (27)$$

where

$$\nabla . \boldsymbol{F} = R, \quad \nabla . \boldsymbol{S} = Q. \qquad (28a, b)$$

It is more direct to define Q than the corresponding flux $\boldsymbol{S}$. Furthermore, R can be determined directly from the specific intensity of radiation, at least in principle. Therefore, it appears more natural and simple to adopt the first formulation. The coefficient of ∇T^* then consists only of the variation of the thermal conductivity.

4. ANALYSIS

The discussion of the stability of T_s has now been converted to the study of the general properties of the perturbation equation (22). Since the time-dependence of the solution of equation (22) will be the deciding factor in this study the first step should be to separate out the time and space components of the solution by admitting a product form

$$T^*(\boldsymbol{r}, t) = f(\boldsymbol{r}) . \theta(t), \qquad (29)$$

which effects

$$\theta' + \mu\theta = 0, \qquad (30)$$

$$\nabla^2 f + \boldsymbol{\beta} . \nabla f + [\gamma + (\mu/\alpha)]f = 0, \qquad (31)$$

where μ is the separation constant, a real number. The perturbation temperature T^* will be assumed to satisfy homogeneous boundary conditions. This assumption is not a restriction from the stability viewpoint.

Clearly, the solution contains an exponential time factor $\theta = e^{-\mu t}$. The initial perturbation should, therefore, be expressible in terms of the solutions of (31), $f(\boldsymbol{r}; \mu)$. Depending on the finite or infinite extent of the boundaries, the representation may either be of series or integral or mixed type. The summation or integration parameters contain discrete or continuous eigenvalues of μ. Since (31) is multidimensional, the eigenvalues of μ are, in general, coupled with the eigenvalues of the other separation parameters if equation (31) is separable. Otherwise, the problem is too complicated to be considered here.

In order that any arbitrary initial perturbation $T^*(\boldsymbol{r}, 0)$ be expressible in terms of the functions $f(\boldsymbol{r}; \mu)$ the latter must be a complete set. The set of all eigenfunctions of equation (31), if they exist, is of this character, namely complete. Therefore, one should consider all eigenvalues of (31). Among them μ is the one with a decisive role on stability since it appears as the time coefficient in the exponential factor. If the eigenvalues of μ are all positive then all perturbations are stable. If, on the other hand, the set of eigenvalues consists of negative members the perturbations which are expressible in terms of the eigenfunctions corresponding to the negative eigenvalues are unstable. This means that the steady temperature will grow anyway. Finally, if $\mu = 0$ is an eigenvalue of (31) then, since (30) gives a time factor proportional to time, the temperature is again unstable.

Further separation of equation (31) into its coordinate components is, in general, not possible. However, it may be simplified in form by a proper choice of coordinate axes at the point of interest. Hence, for example in local rectangular coordinates with Ox-axis parallel to $\boldsymbol{\beta}$, the spatial component (31) of the perturbation equation becomes

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} + \beta \frac{\partial f}{\partial x} + \left(\gamma + \frac{\mu}{\alpha}\right) f = 0, \quad (32)$$

where β is the magnitude of $\boldsymbol{\beta}$. Equation (32) cannot be separated still since β and γ are generally functions of all three coordinates.

Leaving the question whether equation (32) possesses a complete set of characteristic solutions for f with a lowest eigenvalue of μ, it may be reiterated that if it does then the steady-state temperature is stable only if the lowest eigenvalue of μ is positive. If the range of eigenvalues of μ consists of nonpositive values there are then certain preferred forms of perturbation which are unstable and expressible in terms of the set of eigenfunctions corresponding to those nonpositive eigenvalues.

5. SPECIAL CASE: THE STEADY PROBLEM IS ONE-DIMENSIONAL

In order to be able to reach more concrete conclusions the problem will now be restricted to the case where α, β and γ are functions of x alone. In view of the structure of these quantities, the assumption requires that the steady parameters be one-dimensional. The perturbation, however, is still allowed to vary three-dimensionally.

Introduction of a product solution $f = X(x)Y(y)Z(z)$ into equation (32) results in three equations,

$$\frac{d^2X}{dx^2} + \beta(x)\frac{dX}{dx} + \left[\gamma(x) + \frac{\mu}{\alpha(x)} - (k + l)\right]X = 0, \quad (33)$$

$$\frac{d^2Y}{dy^2} + kY = 0, \quad (34)$$

$$\frac{d^2Z}{dz^2} + lZ = 0. \quad (35)$$

The last three equations are all special forms of Liouville equation

$$\frac{d}{dz}\left[p(z)\frac{dy}{dz}\right] + [q(z) + \Lambda r(z)]y = 0. \quad (36)$$

The general behaviour of the solutions of (36) is determined by the functions p, q, r and the parameter Λ. The important results of the analysis of Liouville equation are the following (see e.g. [7], Chapters 5 and 6).

Let p, q, r be nonsingular and p, r positive within a finite interval of z.

(a) There is an infinite set of district values (eigenvalues) of Λ for which the solutions (eigenfunctions) are oscillatory, and they satisfy Dirichlet–Neumann type boundary conditions.

(b) The set of eigenfunctions is complete.

(c) The set of eigenvalues contain a minimum Λ_{min} and extends to infinity.

(d) An oscillatory solution is obtained if $q + \Lambda r > 0$. The condition sets a lower bound $-q/r$ for eigenvalues, i.e. $\Lambda_n > -(q/r)$, but does not determine Λ_{min}.

(e) If the interval for z extends to infinity the distribution of eigenvalues and eigenfunctions are continuous rather than being discrete. In this case, Λ_{min} tends to the lower bound $-q/r$.

For stability purposes one may consider an infinite medium thus allowing the temperature perturbations to span the whole space. A continuous spectrum of eigenvalues and complete set of eigenfunctions are then needed to represent any possible three-dimensional perturbation. Returning to equations (33)–(35), the allowed range of eigenvalues should coincide with the interval $-(q/r) < \Lambda \leqslant \infty$ in each of them. In equations (34) and (35) $p = r = 1, q = 0$, hence $k > 0, l > 0$. In equation (33), on the other hand, $p = \mathrm{e}^{\int \beta \mathrm{d}x}$, $q = p[\gamma - (k + l)]$, and $r = (p/\alpha)$. Obviously, p and r are positive and nonsingular. Supposing that all gradients at steady-state are finite, q has no singularity either; the eigenvalues all satisfy the condition

$$\mu > -\alpha[\gamma - (k + l)]. \tag{38}$$

For stability μ should be positive. Since its smallest value is obtained when k and l are smallest, namely zero, one concludes that the perturbations are stable if γ is negative. In terms of physical parameters the criterion for stability is then

$$\frac{\mathrm{d}}{\mathrm{d}x}(\ln \kappa)'_s \frac{\mathrm{d}T_s}{\mathrm{d}x} + \left(\frac{Q}{\kappa}\right)'_s - \left(\frac{R}{\kappa}\right)'_s < 0. \tag{39}$$

It is equally possible to consider a locally one-dimensional steady case which has cylindrical or spherical symmetry instead. Separation is again accomplished in the same lines, but now α, β, and γ depend upon x, the radial coordinate, alone. It can be shown by similar arguments that the stability criterion (39) is valid in those cases too.

6. DISCUSSION

(a) The condition for stability was obtained in a special case: The steady problem is locally one-dimensional, may the nontrivial dimension be planar, cylindrical, or spherical at the point of interest. Usually the dependence of the physical properties on coordinates as well as on temperature is rather weak. It is, therefore, tempting to generalize the criterion to the cases where the steady problem also is three-dimensional. At least when the properties vary slowly one may expect that stability exists if

$$\nabla(\ln \kappa)'_s \,.\, \nabla T_s + \left(\frac{Q}{\kappa}\right)'_s - \left(\frac{R}{\kappa}\right)'_s < 0. \tag{40}$$

(b) The effect of the first term of (39) on stability may either be like that of a source or a loss. Writing explicitly,

$$\frac{\mathrm{d}}{\mathrm{d}x}(\ln \kappa)'_s \frac{\mathrm{d}T_s}{\mathrm{d}x} = \frac{1}{\kappa_s}\frac{\mathrm{d}\kappa'_s}{\mathrm{d}x}\cdot\frac{\mathrm{d}T_s}{\mathrm{d}x} - \frac{\kappa'_s}{\kappa_s^2}\frac{\mathrm{d}\kappa_s}{\mathrm{d}x}\cdot\frac{\mathrm{d}T_s}{\mathrm{d}x}, \tag{41}$$

it is seen that the first term on the right behaves like an additional energy source if κ'_s and T_s both increase in the same direction, and like a loss if they increase in opposite directions. When κ'_s is positive the second term acts like a loss term if κ_s and T_s increase in the same direction, and like a source term otherwise. When it is negative the reverse is true.

In particular, if the thermal conductivity is independent of temperature the stability criterion simplifies to

$$\left(\frac{Q}{\kappa}\right)'_s < \left(\frac{R}{\kappa}\right)'_s. \tag{42}$$

(c) In the majority of cases of interest, the physical properties may be assumed constant, i.e. independent of temperature and coordinates. Then, (40) reduces to

$$Q'_s < R'_s, \tag{43}$$

meaning that the loss term should depend upon temperature more strongly than the source term does.

(d) In certain phenomena of physics governed by the general diffusion equation (18) the stability of the physical quantity in question may be checked by the same criterion. In [7] may be found several examples of interest scattered throughout the text (see Ch. 12.1, pp. 1595–1603 for an application to chain reaction in fissionable material).

(e) A final remark should be made about the assumption that eigenvalues and eigenfunctions are distributed continuously. The example given at the outset shows that, in order for the medium to be considered as infinite, its boundaries should be separated by a distance which is large compared to a characteristic length. In the general case, one should be able to make a similar comparison with a suitably defined characteristic length. If the characteristic dimension of the medium is comparable with the characteristic length then the lowest eigenvalue may considerably be greater than the lower bound $-q/r$; hence, for stability q need not be negative. It may have a small but finite positive value. The roles of the quantities Q and R, however, do not change although they should now be evaluated for a system with finite boundaries. Nevertheless, the assumption of infinite medium gives a more general condition for stability which is always valid.

7. CONCLUSION

The analysis provides a stability criterion which is applicable to any locally one-dimensional, steady temperature distribution, either plane or with cylindrical or spherical symmetry. It is also applicable to other diffusion problems provided that the physical parameters be interpreted anew.

It is important that, although the steady state is assumed to be one-dimensional, perturbations are not limited to one dimension only; they may be three-dimensional and may be represented in product form through series or integrals over the spectrums of eigenvalues.

In the general, three-dimensional case the counterpart of condition (39) must be sought before it is generalized into (40). This requires a thorough analysis of equation (31).

REFERENCES

1. I. M. Winer, Solution to a class of complete heat flow equations, *Q. Appl. Math.* **23**, 82 (1965).
2. N. Y. Ölçer, Note on the general solution of the heat equation, *Q. Appl. Math.* **24**, 380–383 (1967).
3. N. Y. Ölçer, General solutions to a class of heat flow problems in a finite circular cylinder, *Br. J. Appl. Phys.* **18**, 89–105 (1967).
4. J. H. Lynch, Two new theorems in heat conduction, *Int. J. Heat Mass Transfer* **11**, 917–924 (1968).
5. D. F. Hays and H. N. Curd, Heat conduction in solids: Temperature dependent thermal conductivity, *Int. J. Heat Mass Transfer* **11**, 285–295 (1968).
6. B. M. Cohen, Heat transfer by conduction and radiation with temperature-dependent thermal conductivity. *J. Heat Transfer* **91**, 150–160 (1969).
7. P. M. Morse and H. Feshbach, *Methods of Theoretical Physics*, Chapter 12.1, pp. 1596–1599. McGraw-Hill, New York (1953).

SUR LA STABILITÉ DE LA DISTRIBUTION DANS UN SOLIDE TRANSPARENT AVEC UNE SOURCE INTERNE D'ÉNERGIE

Résumé—On étudie la stabilité de la solution de l'équation générale de la conduction de chaleur avec source interne d'énergie fonction de la température et de la position, rayonnement et propriétés physiques variables. On montre à partir d'un exemple simple que la source d'énergie dépendant de la température peut provoquer un accroissement illimité de la température. Dans le cas de distributions monodimensionnelles stationnaires de température et de propriétés on obtient la condition de stabilité de la distribution de température dans un solide infini. Le résultat est discuté ainsi que sa validité dans différents cas.

DIE STABILITÄT DER STATIONÄREN TEMPERATURVERTEILUNG IN EINEM DURCHSICHTIGEN FESTKÖRPER MIT INNEREN WÄRMEQUELLEN

Zusammenfassung—Die Stabilität der Lösung der allgemeinen Wärmeleitungsgleichung mit temperaturabhängiger-und lageabhängiger innerer Wärmeerzeugung, Strahlung und abhängigen Stoffwerten wird untersucht. An einem einfachen Beispiel wird gezeigt, dass eine temperaturabhängige Wärmequelle eine unbegrenzte Temperaturerhöhung hervorrufen kann. Für eindimensionale, stationäre Temperatur- und Stoffwertverteilung wird die Bedingung für stabile Temperaturverteilung im unendlichen Körper abgeleitet. Die Gültigkeit des Ergebnisses wird für verschiedene Fälle diskutiert.

ОБ УСТОЙЧИВОСТИ СТАЦИОНАРНОГО РАСПРЕДЕЛЕНИЯ ТЕМПЕРАТУРЫ В ПРОЗРАЧНОМ ТВЕРДОМ ТЕЛЕ С ВНУТРЕННИМ ИСТОЧНИКОМ ЭНЕРГИИ

Аннотация—Изучается стационарное решение уравнения теплопроводности в случае, когда внутренний источник энергии, излучение и физические свойства зависят как от температуры, так и от координат. На простом примере показано, что источник энергии, зависящий от температуры, может вызвать беспредельный рост температуры. В случае одномерного стационарного распределения температуры и свойств получено условие устойчивости распределения температуры в бесконечном твердом теле. Обсуждается справедливость полученных результатов.